Stochastic Optimization Algorithms for Capital Budgeting

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One of the RIAM objectives is to optimize the capital budgeting to support decisions for NPP operations. Since the RIAM are influenced by various factors, such as markets, safety and regulatory, the decision-making process of RIAM should take into account relevant factors for balancing risks, costs and profits. The traditional method of capital budgeting is based on the priority list of candidate projects using economic measures such as benefit-investment ratio, net present value and internal return rate. In the literatures, the problem of capital budgeting or the variant can be represented by an appropriate knapsack problem. The knapsack approach to capital budgeting takes as input as investments, along with the cost and profit of each project. The objective of capital budgeting is to find the combination of the binary decisions for every investment such that the overall profit is as large as possible. The output is a collection of projects to be carried out, and we refer this selected collection of projects as a project portfolio. However, as it is frequently the case for capital budgeting with NPP applications, in practice several additional constraints, such as resources/liabilities, dependencies/synergies, options, time windows for every investment etc., have to be fulfilled. This leads to a various extensions and variations of the basic knapsack problem. Because this need for extension of the basic knapsack problem arose in many practical applications, we will present several more general variants of knapsack problem and their implementations in the following sections.

# Risk-Free Decision Making for Capital Budgeting

If the costs and profits of the candidate projects as well as the budgets are known with certainty, the knapsack model provides an effective tool for selecting a project portfolio. The basic knapsack problem (KP) for capital budgeting can be defined as follows: We are given an instance of the capital budgeting problem with investment set , consisting of investments with profit , e.g. net present value (NPV), and cost , and the available budget . Then the objective is to select a subset of such that the total profit of the selected investments is maximized and total cost does not exceed . Alternatively, this (KP) can be formulated as a solution of the following linear integer programming formulation:

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| --- | --- |
|  | (KP-1) |

|  |  |
| --- | --- |
|  | (KP-2) |

|  |  |
| --- | --- |
|  | (KP-3) |

Constraint in (KP-2) ensures that the cost of the project portfolio is within the budget. The binary variables in (KP-3) which correspond to the selection in the binary decision (1 if project is selected; 0 otherwise). This is the simplest non-trivial integer programming model with binary variables. The variants and extensions of the (KP) will be treated in following sub-sections.

## Bounded Knapsack Problem

In the capital budgeting problem described above it may be the case that not all investments/projects are different from each other. In particular, in practice there may be given a number of identical pumps/valves to be replaced. In this case the number of decision variables is equal to the number of different investments instead of the total number of investments. Formally, constraint (KP-3) is replaced by non-negative integer decision variable

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The resulting problem is called the bounded knapsack problem (BKP) formally defined by

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| --- | --- |
|  | (BKP-1) |

|  |  |
| --- | --- |
|  | (BKP-2) |

|  |  |
| --- | --- |
|  | (BKP-3) |

## Multi-dimensional knapsack problem or D-dimensional knapsack problem (DKP)

Moving in a different direction, we consider again the basic capital budgeting problem, i.e. (KP-1) ~ (KP-3), and now take into account not only the cost constraint but also the limited commitment of critical resources, including: (i) capital cost, (ii) operation and maintenance costs, (iii) time and labor-hours during a planned outage, (iv) personnel, installation and maintenance equipment, space, and more. Denoting the cost of every investment by for each resource and introduce the corresponding limited resource we can formulate the extended capital budgeting problem by replacing constraint (KP-2) in (KP) by:

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The resulting problem is called multi-dimensional knapsack problem or D-dimensional knapsack problem (DKP) formally defined by:

|  |  |
| --- | --- |
|  | (DKP-1) |

|  |  |
| --- | --- |
|  | (DKP-2) |

|  |  |
| --- | --- |
|  | (DKP-3) |

Where the limited resources set is denoted by , consisting of “colors” of money within capital costs, within operation and maintenance costs, within personnel availability, etc.

Another example is that the plant has multi-year investments. Consider a DKP problem in which the costs of each investment and the available capitals vary according to time period . By defining as the cost of investment at time period , and as the available capital at time period , we get:

## Multiple Knapsack Problem (MKP)

Another interesting variant of the capital budgeting problem arises from the original version described above if we consider a maintenance for multi-units NPP in parallel, i.e. it has to be decided whether to accept a particular replacement and in the positive case in which unit to conduct the corresponding replacement. This can be formulated by introducing a binary decision variable for every combination of a maintenance with a unit. If there are investments (investment set ) on the list of maintenance requests and unit (unit set ) available, we use binary variables:

The resulting problem is called the multiple knapsack problem (MKP), and the mathematical formulation is given by

|  |  |
| --- | --- |
|  | (MKP-1) |

|  |  |
| --- | --- |
|  | (MKP-2) |

|  |  |
| --- | --- |
|  | (MKP-3) |

|  |  |
| --- | --- |
|  | (MKP-4) |

## Multiple-choice knapsack problem

Another quite different variant of the capital budgeting problem appears if there may be multiple ways to carry out each investment/project. Each investment however exists in a number of options where the *j-th* option has cost and profit value . This problem may be expressed as the multiple-choice knapsack problem (MCKP). Assume is the set of different options of investment . Using the decision variables to denote whether option was chosen from the set , the mathematical formulation of MCKP is given by

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| --- | --- |
|  | (MCKP-1) |

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| --- | --- |
|  | (MCKP-2) |

|  |  |
| --- | --- |
|  | (MCKP-3) |

|  |  |
| --- | --- |
|  | (MCKP-4) |

Constraint (MCKP-3) ensures that exactly one option is chosen from each investment. Considering the limited resources and multi-year investments mentioned in section 15.1.2, the MCKP may be extended to D-dimensional MCKP problem (D-MCKP). For example, a project may be performed over a three-year period, say, years , or the start of the project could instead be two years hence with project implementation over years . Alternatively, at increased cost and increased benefit, it may be possible to complete the project in two years, or . When selecting a project to uprate plant capacity, we may have two options that increase capacity by 3% or 6%. In these cases, the problem can be expressed as the D-MCKP. This problem is formally defined as follows

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# Risk-Hedge Decision Making: Prioritizing Project Selection

One limitation of traditional optimization models for capital budgeting is that they do not account for risk/uncertainty in profit and cost streams associated with individual projects, they do not account for risk in resource availability in future years. Projects can incur cost over-runs, especially when projects are large, performed infrequently, and when there is risk regarding technical viability, external contractors, and/or suppliers of requisite parts and materials. Occasionally, projects are performed ahead of schedule and with cost savings. Planned budgets for capital improvements can be cut and key personnel may be lost. Or, there may be surprise windfalls in budgets for maintenance activities due to decreased costs for “unplanned” maintenance. In these cases, how should we resolve capital budgeting when we have risk forecasts for costs, profits and budgets? One approach is to re-solve the models described in section 15.1 when refined forecasts for these parameters become available. However, it is not always practical to fully revise a project portfolio whenever better forecasts become available.

In order to prioritize the project selection with risk forecast for these parameters, the two-stage stochastic optimization model [REF] is employed to provide priority lists to decision-makers to support better risk-informed decisions. Its inputs include those described in section 15.1 for different variant of the capital budgeting problem, except that a probabilistic description of the uncertain parameters is integrated in the optimization process. The two-stage stochastic optimization model forms a priority list as its first-stage decision and then forms a corresponding project portfolio for each scenario as its second-stage decision. When forming the optimal second-stage project portfolio under a specific scenario, the stochastic optimization model ensures that the portfolio is consistent with the first-stage prioritization; i.e., a project can be selected only if all high-priority projects are also selected. Thus, the portfolios of projects corresponding to different scenarios are nested.

The risk-free capital budgeting models presented in section 15.1 assumes that there is no risk in the problem data. And, as was demonstrated in [REF], the models do not naturally produce a priority list. The need to deal with these risk forecasts for costs, profits and budget motivates extending risk-free models to risk-informed models that can form a priority list with the goal of maximizing profits of the investments. The notation and formulation of the risk-informed models are as follows:

Indices and Sets:

, candidate projects

, options for selecting project , e.g., initiate project in year or and in a standard

(three year) or in an expedited (two year) manner. Note that the last option for project is always used to indicate “non-selection”, i.e. the investment is not selected.

, types of resources, e.g., capital funds, O&M funds, labor-hours, time during outage

, time periods (years)

, scenarios

*Data:*

= profit of investment under scenario (NPV)

= profit of investment via option under scenario (NPV)

= available budget under scenario 

= available budget for a resource of type under scenario 

= available budget for unit under scenario 

= available budget in year under scenario 

= available budget for a resource of type in year under scenario 

= cost of investment under scenario 

= consumption of resource of type if investment is selected under scenario 

= consumption of resource in year if investment is performed via option under scenario 

= consumption of resource of type in year if investment is performed via option under scenario 

= probability of scenario 

*Decision variables:*

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The risk-free capital budgeting models could be reformulated into risk-informed models as shown in the following:

**(Risk-Informed SKP Model: RI-SKP)**

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| --- | --- |
|  | (RI-SKP-1) |

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|  | (RI-SKP-2) |

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| --- | --- |
|  | (RI-SKP-3) |

|  |  |
| --- | --- |
|  | (RI-SKP-4) |

**(Risk-Informed DKP Mode: RI-DKP)**

|  |  |
| --- | --- |
|  | (RI-DKP-1) |

|  |  |
| --- | --- |
|  | (RI-DKP-2) |

|  |  |
| --- | --- |
|  | (RI-DKP-3) |

|  |  |
| --- | --- |
|  | (RI-DKP-4) |

**(Risk-Informed MKP Model: RI-MKP)**

|  |  |
| --- | --- |
|  | (RI-MKP-1) |

|  |  |
| --- | --- |
|  | (RI-MKP-2) |

|  |  |
| --- | --- |
|  | (RI-MKP-3) |

|  |  |
| --- | --- |
|  | (RI-MKP-4) |

|  |  |
| --- | --- |
|  | (RI-MKP-5) |

**(Risk-Informed MCKP Model: RI-MCKP)**

|  |  |
| --- | --- |
|  | (RI-MCKP-1) |

|  |  |
| --- | --- |
|  | (RI-MCKP-2) |

|  |  |
| --- | --- |
|  | (RI-MCKP-3) |

|  |  |
| --- | --- |
|  | (RI-MCKP-4) |

|  |  |
| --- | --- |
|  | (RI-MCKP-5) |

|  |  |
| --- | --- |
|  | (RI-MCKP-6) |

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|  | (RI-MCKP-7) |

All these models are a two-stage stochastic integer program. The first-stage decision variable, , form the priority list. The second-stage decision variable, , selects the portfolio of projects to implement for each scenario. The objective function (RI-\*-1) captures the expected total profit, forming the weighted sum of profits over all scenarios. Constraint (RI-\*-2) ensures that the implemented investments stay within budget under each scenario, for each year and/or for each resources/liabilities. Given a pair of investments, constraint (RI-\*-3) ensures either they have the same priority or one has higher priority than the other. Constraint (RI-\*-4) requires that the investment selected by , under each scenario, are consistent with the priority list’s ordering. The last constraint, i.e. RI-MKP-5, ensures each investment can be selected at most by one unit, and the constraint, i.e. RI-MCKP-5, requires only one option per investment can be selected. There are two optional constraints for RI-MCKP problem, i.e. RI-MCKP-6 and RI-MCKP-7, that can be enabled to provide more consistent results for decision makers. Constraint RI-MCKP-6 will generate a total ordering of the projects rather than allowing ties, and constraint RI-MCKP-7 is a type of consistency constraint with respect to the notion of options; the constraint matters only when project is higher priority than project . In this case, if we select Plan A for the lower priority project then we must select plan A for the higher priority project. If we select Plan B for the lower priority project then we can select Plan A or Plan B for the higher priority project. And, if we select Plan C for the lower priority project then we can select Plan A, B, or C for the higher priority project. Inclusion of constraint RI-MCKP-7 is “optional” and reflects how the decision maker prefers to interpret the notion of priorities.